On Candidates for Divergence in Natural Collatz Variants

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The Collatz Problem

n -> 3n+1 and down-halfing, repeat

Only one limit cycle: 1 -> 4 -> 2 -> 1

The most natural variant:

n -> 3n-1 and down-halfing

Only three limit cycles:

(1)
(5,7, 5)
(17,25,37,55,41,61,91, 17)

Heuristic argument:

After each *3-operation in average two divisions by 2 follow; hence in average factor 3/4 < 1 in each round.

(Assumption: random properties of 3n+1 , and 3n-1)

Generalisation

We fix real parameters

$$1 < x < 2, \quad y > 0,$$

and define a map $T_{x,y}$ on the **odd positive integers** by

1. Start with an odd n.
2. Compute $t = xn + y$.
3. Let $u = \lfloor t \rfloor \in \mathbb{Z}$.
4. Factor out all powers of 2:

   $$u = 2^k m, \quad m \text{ odd}.$$

5. Set

   $$T_{x,y}(n) := m.$$

The iteration is

$$n_{k+1} = T_{x,y}(n_k), \quad n_0 \text{ odd}.$$

* The case with $x = 3/2$ and $y = 1$ corresponds to the $3n+1$ problem

* The case with $x = 3/2$ and $y = 0$ corresponds to the $3n-1$ problem.

Heuristic: After each $*x$-operation there is in average one division by 2; hence in average factor $x/2 < 1$ in each round

(Assumption: random properties of downrounded $xn+y$)

Does not hold for x = 4/3 and several y, n

4/3 * n + 1   diverges for n=11
4/3 * n + 2.5   diverges for n=5

4/3 * n + 5   diverges for n=1
4/3 * n + 6.5   diverges for n=59

....
In general, it seems
4/3 * n + (4k+1)   diverges for some small n
4/3 * n + (4k+2.5)diverges for some small n

for all natural k (checked until k=9)

Only heuristic arguments, no proofs!

Checking many tuples (x,y;n), we found

interesting sequences for
x near to sqrt(2) and y=4

1
5

11
19*
15
25

39
59
87

127
183*
131
189

271
387
551
783
1111
1575
2231

3159
4471*
3163
4477

6335

8963
12679*
8967
12685

17943
25379
35895
50767
71799

101543
143607*
101547
143613

203103

287235
406215*
287239
406221

574487
812451
1148983
1624911

2297975
3249831

4595959
6499671*
4595963
6499677

9191935
12999363
18383879

25998735
36767767*
25998739
36767773

So smooth, but how to prove?

ChatGPT-5 did the job of proving!

We told it the recursion and our observations in the sequence above. Then we asked: Now prove this!

Part 1: For starting value $n(0)=1$ and each iteration t the sequence satisfies either $n(t+1) > n(t)$ or $n(t+2) > n(t)$.

Part 2: for every odd starting value after some initial segment we get for all $t > T$: either $n(t+1) > n(t)$ or $n(t+2) > n(t)$.

Making Mathematics with the Help of AI

Using ChatGPT and other Large Language Modells (LLM) in Math for:

* Quick reading, structuring and abstracting of papers (moving pdf with drag and drop)

* Error search in papers (own ones or others)

* Performing small or medium-size computations to collect data (without classical programming)

* "little proofs" (Terence Tao's experiences: to save some hours of own working time)

* Using an LLM-AI like GPT-5 for searching data bases such **OEIS,**
finding structures and formulating conjectures

* LLM in teamwork with a theorem prover, for instance **LEAN**

A few random hints and remarks

* important when using GPT-5: use strict mathematical mode (& "Thinking mode")

* GPT-5 account for 23 Euro per month

* very nice with GPT-5: during the thinking process current thoughts are shown in grey script, changing every fraction of a second. These intermediate thoughts are not protocolled! But they will help your intuition.

* Problems: sometimes (or often; depending on patience) LLMs have serious logical faults concerning statements which are elementary for humans.

** History: Chess with Computer Help **

Karsten Müller (born 1970; Chess Grandmaster and
PhD in Mathematics) was one of the early adoptors of
chess programs to analyse positions (in 1992/1993).

2008: Karsten Müller (left), Chrilly Donninger (right), Jakob Erdmann (back)

Chess program **Shredder** in 4-best analysis mode, version Shredder-3 from 1998 (author **Stefan Meyer-Kahlen**). See this site for more information on Shredder.

The company **ChessBase.com** (seated in Hamburg) produces and sells software for computer-aided chess analysis since 1987. World Champion **Garry Kasparov** was one of their very first power users.

Preprints

(currently my personal favorites)

What is the point of computers? A question for pure mathematicians
Kevin Buzzard. April 18, 2022.

Mathematical proof between generations
Jonas Bayer, ... Kevin Buzzard, Marco David, Leslie Lamport, Yuri Matiyasevich ... July 2022.

O-Forge: An LLM + Computer Algebra Framework for Asymptotic Analysis
Ayush Khaitan and Vijay Ganesh. October 14, 2025.

Forbidden Sidon subsets of perfect difference sets, featuring a human-assisted proof
Boris Alexeev and Dustin G. Mixon. October 22, 2025.

Mathematical exploration and discovery at scale
Bogdan Georgiev, Javier Gomez-Serrano, Terence Tao, and Adam-Zsolt Wagner. November 03, 2025.

New Nikodym set constructions over finite fields
Terence Tao. November 11, 2025

Disclaimer: This is my very personal list. It will change over time.
Other mathematicans have other favorites.

Some screenshots with marks from the arxiv preprint 2511.07721, highlighting interactive use of AI tools by Terence Tao

11 Nov 2025

NEW NIKODYM SET CONSTRUCTIONS OVER FINITE FIELDS

TERENCE TAO

ABSTRACT. For any fixed dimension $d \geq 3$ we construct a Nikodym set in $\mathbf{F}_q^d$ of cardinality $q^d - (\frac{d-2}{\log 2} + 1 + o(1))q^{d-1}\log q$ in the limit $q \to \infty$, when q is an odd prime power. This improves upon the naive random construction, which gives a set of cardinality $q^d - (d-1+o(1))q^{d-1}\log q$, and is new in the regime where $\mathbf{F}_q$ has unbounded characteristic and q not a perfect square. While the final proofs are completely human generated, the initial ideas of the construction were inspired by output from the tools `AlphaEvolve` and `DeepThink`. We also give a new construction of Nikodym sets in $\mathbf{F}_q^2$ for q a perfect square that match the existing bounds of $q^2 - q^{3/2} + O(q \log q)$, assuming that q is not the square of a prime $p \equiv 3 \pmod 4$.

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see Section 2.1. However, this bound was only heuristic.

- After several failed attempts (by both the human authors and `DeepThink`) to make this heuristic precise, a weaker bound

$$\mathrm{Nikodym}(d,q) \leq q^d - \left(\frac{d-2}{\log 2} + 1 + o(1)\right) q^{d-1}\log q \tag{1.9}$$

was established for odd q, which improved upon (1.7) when $d \geq 3$. (For even q, the

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This construction was initially found by `AlphaEvolve` and verified by `DeepThink`, but the proof we give in Section 4 is human-written.

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As discussed below, a more complicated variant of this method was first discovered by `Deepthink`. After suggesting a purely random construction, `Deepthink` was also able to reconstruct most of the details of the above argument.

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one could increase k to approximately $\log(q^{d-1})/log2$, thus predicting the bound (1.8).

An initial attempt to use `Deepthink` to reproduce these heuristics was unsuccessful, and identified a geometric flaw in the construction: as quadratic polynomials of one variable will usu-

Two screenshots with marks from the arxiv preprint 2510.12350v2 by A. Khaitan and V. Ganesh.

O-Forge: An LLM + Computer Algebra Framework for Asymptotic Analysis

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Abstract

Large language models have recently demonstrated advanced capabilities in solving IMO and Putnam problems; yet their role in research mathematics has remained fairly limited. The key difficulty is verification: suggested proofs may look plausible, but cannot be trusted without rigorous checking. We present a framework, called LLM+CAS, and an associated tool, O-Forge, that couples frontier LLMs with a computer algebra systems (CAS) in an In-Context Symbolic Feedback loop to produce proofs that are both creative and symbolically verified. Our focus is on asymptotic inequalities, a topic that often involves difficult proofs and appropriate decomposition of the domain into the "right" subdomains. Many mathematicians, including Terry Tao, have suggested that using AI tools to find the right decompositions can be very useful for research-level asymptotic analysis. In this paper, we show that our framework LLM+CAS turns out to be remarkably effective at proposing such decompositions via a combination of a frontier LLM and a CAS. More precisely, we use an LLM to suggest domain decomposition, and a CAS (such as Mathematica) that provides a verification of each piece axiomatically. Using this loop, we answer a question posed by Terence Tao: whether LLMs coupled with a verifier can be used to help prove intricate asymptotic inequalities. More broadly, we show how AI can move beyond contest math towards research-level tools for professional mathematicians.

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Our primary novelty is in being able to automate proof completion for difficult research problems that should take most research mathematicians lots of time and effort. No existing AI tools are able to complete and symbolically verify proofs of this kind. Moreover, although frontier LLMs may be able to produce some of these proofs, these proofs are often incorrect, and need to be manually verified. Our tool does away with the need for manual verification.

2 Acknowledgements

We gratefully acknowledge Terence Tao for repeatedly stress-testing our tool and suggesting substantial improvements to our website, `o-forge.com`. We also thank Swarat Chaudhuri, Aiman Kohli, Amitayush Thakur, and George Tsoukalas for their valuable help and advice.

My website on Collatz Prizes

https://althofer.de/collatz-prizes.html

From time to time I get proposals for solutions from hobby mathematicians.

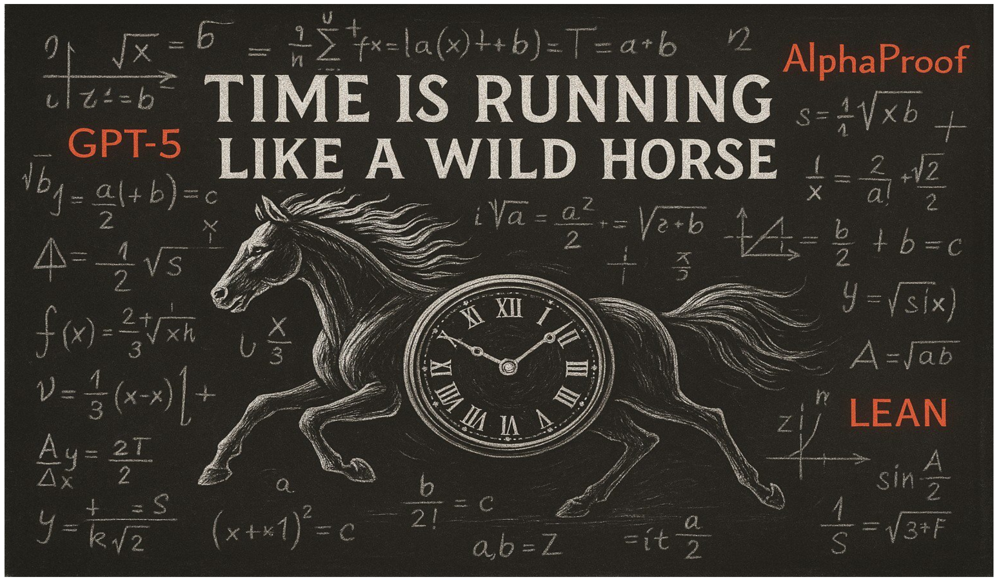
Often a quick check by GPT-5 together with a control look by me is enough to identify serious errors.

This typically saves me a lot of time. Of course it would be better if afficionados “checked” their proofs with the help of GPT-5 or Gemini 2.5 before submitting them.

Thanks to Karsten Müller, Dietmar Wolz, Ulrich Tamm,
Torsten Sillke, Michael Taktikos, and Heinrich Burger
(the first top correspondence chess player to use intensive computer help in 1988)

for collaboration, discussions and encouragement.

https://althofer.de



Time is running like a wild horse.
Picture by ChatGPT-5, prompt by I.A. Nov 13 2025.

A typical LLM problem: GPT-5 refuses to put the clock hands on 3 and 14 ;-)